SIDDARTHA INSTITUTE OF SCIENCE AND TECHNOLOGY :: PUTTUR

Siddharth Nagar, Narayanavanam Road – 517583

QUESTION BANK (DESCRIPTIVE)

Subject with Code : Algebra and Calculus **(19HS0830)** **Course & Branch**: B.Tech – Common to al

Year & Sem: I-I **Regulation:** R19

UNIT –I (MATRICES)

1. a) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ into Echelon form and find its rank? [6M]

   b) Find the Eigen value and Eigen vectors of the matrix $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$ [6M]

2. a) Define the rank of the Matrix. [2M]

   b) Find whether the following equations are consistent if so solve them $x + y + 2z = 4\,; 2x - y + 3z = 9\,; 3x - y - z = 2.$ [10M]

3. a) Reduce the matrix A= $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form and find its rank? [6M]

   b) Determine the Eigen values of A^{-1} *where* $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. [6M]

4. a) Solve completely the system of equations x+2y+3z=0 ,3x+4y+4z=0,7x+10y+12z=0. [6M]

   b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [6M]

5. a) State Cayley-Hamilton theorem [2M]

   b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and find A^{-1} ? [10M]

6. Find the Eigen values and corresponding Eigen vectors of the matrix A and its inverse where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. [12M]

7. Diagonalise the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^4 ? [12M]

8) a) Discuss the nature of the Quadratic form $2x_1x_2 + 2x_1x_3 + 2x_2x_3$. [6M]

   b) Identify the nature of the Quadratic form $-3x_1^{\,2} - 3x_2^{\,2} - 3x_3^{\,2} - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$. [6M]

9. Reduce the Quadratic form $3x_1^{\,2}+3x_2^{\,2}+3x_3^{\,2}+2x_1x_2+2x_1x_3-2x_2x_3$ into canonical form by Orthogonal transformation and Find the Rank, Index and Signature of the canonical form. [12M]

10. Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$ into the canonical form by Orthogonal transformation and discuss its nature. [12M]

UNIT –II (CALCULUS AND MEAN VALUE THEOREMS)

1) a) Verify Rolle's Theorem for the function f(x) = $\frac{\sin x}{e^x}$ in $(0,\pi)$. [6M]

b) Verify Lagrange's mean value theorem for f(x) = $\log_e x$ in $[1,e]$. [6M]

2) a) Verify Cauchy's mean value theorem for $f(x) = sinx\ and\ g(x) = cosx\ in\ \left[0,\frac{\pi}{2}\right]$. [6M]

b) Verify Lagrange's Mean value theorem for the functions $f(x) = x(x-1)(x-2) in \left[0,\frac{1}{2}\right]$. [6M]

3) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem. [12M]

4) a) State and verify Rolle's Theorem for the function f(x) =log$\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$ (x$\neq 0$) . [6M]

b) Verify Lagrange's mean value theorem for f(x) =$x^3 - x^2 - 5x + 3$ in $[0,4]$. [6M]

5) a) Verify Cauchy's Mean value theorem for the functions $f(x) = x^3; g(x) = x^2\ in\ [1,2]$ [6M]

b) Express the polynomial $2x^3 + 7x^2 + x$-6 in power of $(x-2)$ assigning Taylor's series. [6M]

6) a) Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem. [6M]

b) Expand $\log_e x$ in powers of (x-1) and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's theorem. [6M]

7) a) Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for log (sec x). [6M]

b) Verify the Rolle's Theorem can be applied to the function f(x) = $\tan x$ in [0,π] [6M]

8) a) Verify Cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in [a, b]. [6M]

b) Show that for any $x > 0,\ 1 + x < e^x < 1 + xe^x$ using Lagrange's mean value theorem. [6M]

9) a) Verify Rolle's theorem for the function f(x) = x(x+3)$e^{-\frac{x}{2}}$ in [-3,0] [6M]

b) Expand $\sin x$ powers of (x- $\frac{\pi}{2}$) up to the term containing $(x - \frac{\pi}{2})^4$ assigning Taylor's series. [6M]

10) Obtain the Maclaurin's series expression of the following functions: [12M]

i) e^x ii) $\cos x$ iii) $\sin x$

UNIT –III (MULTIVARIABLE CALCULUS)

1) a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & at\ (0,0) \end{cases}$ [6M]

b) If $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2+y^2+z^2 \neq 0$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. [6M]

2) a) If $u = tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$,prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. [6M]

b) If $U = log(x^3+y^3+z^3-3xyz)\ prove\ that\ \left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(X+Y+Z)^2}$. [6M]

3) a) Find $\frac{du}{dt}\ as\ a\ total\ derivative; if\ u = x^2y^3$ where $x = logt\ \ and\ y = e^t$. [6M]

b) If $z = xy^2 + x^2y\ ; where\ x = at^2, y = 2at,\ find\ \frac{dz}{dt}$ as a total derivative. [6M]

4) a) If $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3\ , then\ show\ that\ \frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$. [6M]

b) If $u = x^2+y^2+z^2\ and\ x = e^{2t}\ , y = e^{2t}cos3t\ , z = e^{2t}sin3t\ , find\ \frac{du}{dt}$ =? [6M]

5) a) If $u = x^2 - 2y;\ v = x+y+z,\ w = x-2y+3z$,then find Jacobian $J\left(\frac{u,v,w}{x,y,z}\right)$. [6M]

b) Verify if $u = 2x - y + 3z\ , v = 2x - y - z,\ w = 2x - y + z$ are functionally dependent and if so, find the relation between them. [6M]

6) a) In $u = x + 3y^2 - z^3\ , v = 4x^2yz,\ w = 2z^2 - xy, evaluate\ \frac{\partial(u,v,w)}{\partial(x,y,z)}\ at\ (1,-1,0)$. [6M]

b) If $u = x\sqrt{(1-y^2)} + y\sqrt{(1-x^2)}\ and\ v = sin^{-1}x + sin^{-1}y\ , then$ show that u, v are functionally dependent. [6M]

7) a) If $u = \frac{x+y}{1-xy}\ and\ v = tan^{-1}x + tan^{-1}y\ , find\ \frac{\partial(u,v)}{\partial(x,y)}$? [6M]

b) Find the Maximum and Minimum values of $f(x,y) = x^3+y^3-3axy$. [6M]

8) a) Examine the function for extreme values $f(x,y) = x^4+y^4-2x^2+4xy-2y^2$;(x>0,y>0). [6M]

b) Find the stationary points of $u(x,y) = sinx.siny.sin(x+y)\ where\ 0 < x < \pi, 0 < y < \pi$ and find the maximum of u. [6M]

9) a) Find the shortest distance from origin to the surface $xyz^2 = 2$. [6M]

b) Find the minimum value of $x^2+y^2+z^2\ given\ x+y+z = 3a$. [6M]

10) a) Find a point on the plane $3x+2y+z-12 = 0$, which is nearest to the origin. [6M]

b) Find the shortest and longest distance from the point $(3,1,-1)\ to\ the\ sphere\ x^2+y^2+z^2 = 4$.[6M]

UNIT –IV (INTEGRAL CALCULUS)

1. a) Evaluate the following improper integrals i) $\int_1^\infty \frac{1}{x^4} dx$. ii) $\int_0^1 \frac{1}{\sqrt{x}} dx$. [6M]

   b) Prove that $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \pi$. [6M]

2. a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$ [6M]

   b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. [6M]

3. a) Evaluate $\iint (x^2+y^2)dxdy$ in the positive quadrant for which $x+y \leq 1$. [6M]

   b) Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$. [6M]

4. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$ [6M]

   b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by converting to polar coordinates. [6M]

5. a) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. [6M]

   b) Evaluate the integral by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}dxdy$. [6M]

6. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dxdy$. [6M]

   b) Evaluate the integral by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$. [6M]

7. Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} (xy)dydx$ and hence evaluate the same. [12M]

8. a) By changing order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$. [6M]

   b) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$ [6M]

9. a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [6M]

   b) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$. [6M]

10. a) Calculate the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = a$ and $z = 0$. [6M]

    b) Evaluate the triple integral $\iiint xy^2 zdxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [6M]

UNIT –V (SPECIAL FUNCTIONS)

1 a) Define Beta and Gamma functions and Prove that $\Gamma(1) = 1$. [6M]

b) Evaluate $\int_0^1 x^2 \left(\log\frac{1}{x}\right)^3$. [6M]

2. a) Prove that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. [6M]

b) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5} B\left(\frac{2}{5}, \frac{1}{2}\right)$. [6M]

3. a) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^2} dx$. [6M]

b) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}}$ dx = $\frac{1}{2}\ \beta\ (1, \frac{1}{2})$. [6M]

4. a) Evaluate $\int_0^\infty e^{-a^2x^2} dx$. [6M]

b) Express the integral $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ in terms of Beta function [6M]

5. a) Prove that $\int_0^1 (\log\frac{1}{x})^{n-1} dx = \tau(n)$. [6M]

b) Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta . cos^{2n-1}\theta\, d\theta$. [6M]

6. a) Evaluate $\int_0^1 x^4 \left(log\frac{1}{x}\right)^3 dx$. [6M]

b) Prove that $\int_0^1 \sqrt{1-y^4}\, dy = \frac{1}{4}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ [6M]

7. a) Evaluate $\int_0^1 \frac{dx}{\sqrt{-logx}}$ [6M]

b) *Evaluate* $\beta\left(\frac{4}{3}, \frac{5}{3}\right)$ [6M]

8. a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [6M]

b) Prove that $\int_0^{\frac{\pi}{2}} sin^2\theta cos^4\theta d\theta = \frac{\pi}{32}$ [6M]

9. a) Show that $\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6M]

b) Evaluate $\int_0^1 x^3\sqrt{1-x}\, dx$ using β- Γ functions. [6M]

10. Show that $\int_0^1 \frac{x^2\, dx}{\sqrt{1-x^4}}$ x $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{4}$. [12M]